Jan 12 M2 Written Solutions

① (a) $\frac{1}{2}mv^2 = \frac{1}{2} \times 25 \times 60^2 = 45000\text{ J}$

(b) $\Delta GPE = mg\Delta h = 25 \times 9.8 \times 34 = 8330\text{ J}$

(c)(i) No work done against resistive forces so

En gain = GPE loss

$\Rightarrow$ Total En $= 45000 + 8330 = 53330\text{ J}$

(ii) $53330 = \frac{1}{2}mv^2 \quad \Rightarrow v = \sqrt{\frac{2 \times 53330}{25}}$

$v = 65.318 \quad \Rightarrow v = 65.3\text{ ms}^{-1}$

(3sf)

② (a) $F = ma$ so $a = \frac{F}{50}$

$\underline{a} = (6t - \frac{6}{5}t^2)\underline{i} + 2e^{-2t}\underline{j}$

(b) $\underline{v} = \int \underline{a}\, dt = \int (6t - \frac{6}{5}t^2)\underline{i} + 2e^{-2t}\underline{j} \; dt$

so $v = (3t^2 - \frac{2}{5}t^3)\underline{i} - e^{-2t}\underline{j} + \underline{c}$

$\underline{t=0} \quad \underline{v} = 7\underline{i} - 4\underline{j}$ (sub in $t=0$)

$\Rightarrow 0\underline{i} - 1\underline{j} + \underline{c} = 7\underline{i} - 4\underline{j}$

so $\underline{c} = 7\underline{i} - 3\underline{j}$

$v = (3t^2 - \frac{2}{5}t^3 + 7)\underline{i} - (e^{-2t} + 3)\underline{j}$

(c) $t = 1$

$v = (3 - \frac{2}{5} + 7)\underline{i} - (e^{-2} + 3) = 9.6\underline{i} - (3 + e^{-2})\underline{j}$

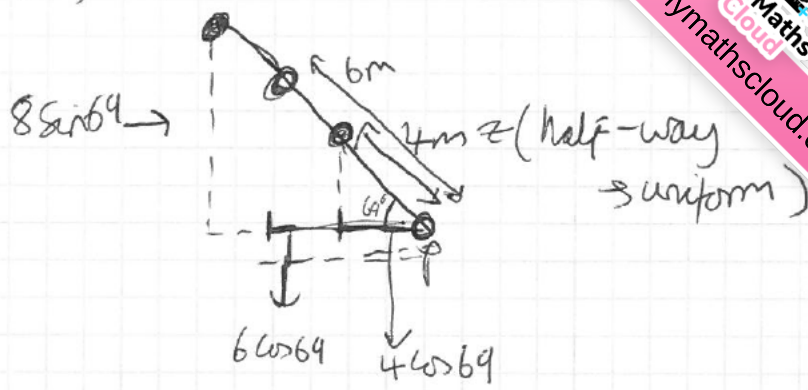
Use Pythagoras to find speed (magnitude of velocity vector)

$\sqrt{9.6^2 + (3 + e^{-2})^2} = 10.099 \qquad$ speed $= 10.1\text{ ms}^{-1}$

(3sf)

3(a)

(b)(i) Take moments abo[ut P]



$$28g \times 4\cos 69 + 72g \times 6\cos 69 = S \times 8\sin 69$$

$$S = \frac{\cos 69\,(28g\times 4 + 72g\times 6)}{8\sin 69} = 255.8\,N$$

$= 256\,N$ (3SF) as required.

b(ii) $S =$ friction (Magnitudes must be equal as horizontal forces must balance)

$R = 72g + 28g$ (vertical forces balanced)

$$\underbrace{S}_{=F} = \mu R$$

so $\mu = \dfrac{255.8}{72g + 28g} = 0.26102$

$\mu = 0.261$ (3SF)

④ (a) Constant speed of $42ms^{-1}$ $\therefore$ Net fo...

$25v \longleftarrow \bullet \longrightarrow F_D$

F_D ↓ driving force

$P_E = F_D v$ ($P = Fv$)

↙ engine power

$\Rightarrow F_D = 25v$

$\frac{P}{v} = 25v$

$P = 25 \times v^2$

$= 25 \times 42^2$

$= 44100\,W$

(as required)

(b) $a \neq 0$ $\therefore$ Net force $\neq 0$

use $F = ma$

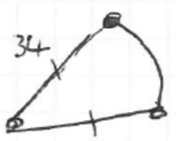
$25v \longleftarrow \bullet \longrightarrow F_D = \frac{P}{v} = \frac{44100}{15}$ ↖ do not use $42ms^{-1}$!

$25v$ $\shortparallel$ 25×15 ↓ not $42 ms^{-1}$!

Net force $= \frac{44100}{15} - 25 \times 15 = 2565\,N$

$F = ma$ $\quad a = \frac{2565}{1500} = \underline{\underline{1.71\ ms^{-2}}}$

⑤



Vertically → forces are balanced so $R = mg$
← (horizontal road)

so $F = \mu R = \mu mg = 0.85mg$

Circular motion: $F = \frac{mv^2}{r}$

so $0.85\cancel{m}g = \frac{\cancel{m}v^2}{34}$

$v = \sqrt{34 \times 0.85 \times 9.8} = 16.829$ $\quad v = 16.8ms^{-1}$ (3SF)

⑥ $F = ma$ $\qquad \frac{dv}{dt} = a = \frac{F}{m}$

$$a = \frac{2}{0.4} - \frac{4v}{0.4} = 5 - 10v \quad = -10v + 5$$

$$= -10(v - 0.5)$$

as required

(b) $$\int \frac{1}{v-0.5}\,dv = -\int 10\,dt$$

$$\ln|v-0.5| = -10t + c$$

so $$e^{-10t+c} = v - 0.5$$

$$e^{-10t}e^{c} = v - 0.5$$

$$Ae^{-10t} = v - 0.5$$

so $$v = \frac{1}{2} + Ae^{-10t}$$

"initial velocity is $1\,ms^{-1}$" so when $t=0$ $v=1$

$$1 = \frac{1}{2} + Ae^{0} \Rightarrow 1 = \frac{1}{2} + A \quad \text{so } A = \frac{1}{2}$$

$$v = \frac{1}{2} + \frac{1}{2}e^{-10t} = \frac{1}{2}\left(1 + e^{-10t}\right)$$

(c) set $v = 0.55$ so $$0.55 = \frac{1}{2}(1 + e^{-10t})$$

$$1.1 = 1 + e^{-10t}$$

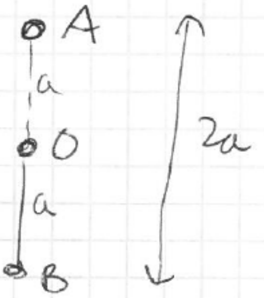
$$\overset{\frac{1}{10}}{0.1} = e^{-10t} \Rightarrow \ln\frac{1}{10} = -10t \Rightarrow t = \frac{-1}{10}\ln\frac{1}{10}$$

$$= \frac{1}{10}\ln 10$$

$$= 0.23026$$

$$t = 0.230\,s$$

7



(a) ~~Energy~~: initial $E_k = \frac{1}{2}mu^2$

some is converted to GPE

$\Delta GPE = mg\Delta h = mg(2a) = 2mga.$

so $E_k = \frac{1}{2}\cancel{m}v^2 = \frac{1}{2}\cancel{m}u^2 - 2\cancel{m}ga$

$(\times 2) \quad v^2 = u^2 - 4ga$ as required.

(b)

use $F = \frac{mv^2}{r}$ $(r = a)$

so at B: $5T - mg = \frac{mu^2}{a}$ ①

A: $2T + mg = \frac{mv^2}{a}$ ②

Eliminate T → so ①×2 and ②×5

$10T - 2mg = \frac{2mu^2}{a}$ ①' and $10T + 5mg = \frac{5mv^2}{a}$ ②'

Do ②' − ①' $\Rightarrow 7\cancel{m}g = \frac{5\cancel{m}v^2}{a} - \frac{2\cancel{m}u^2}{a}$

$7ga = 5v^2 - 2u^2$

use $v^2 = u^2 - 4ga$

$7ga = 5u^2 - 20ga - 2u^2$

$= 3u^2 - 20ga$

$27ga = 3u^2$

$9ga = u^2$

$\Rightarrow u = \sqrt{9ga} = 3\sqrt{ga}$

b(ii) $u : V$

$V^2 = u^2 - 4ag = 9ag - 4ag = 5ag. \quad V = \sqrt{\;}$

$u : V = 3\cancel{\sqrt{ag}} : \sqrt{5}\,\cancel{\sqrt{ag}}$

$3 : \sqrt{5}$ //

(8) (a) $E = \frac{\lambda x^2}{2l} = \frac{\lambda(3-0.8)^2}{2l} = \frac{32 \times 2.2^2}{2 \times 0.8} = \underline{\underline{96.8\,J}}$

(b) EPE at point B $= \frac{\lambda(2-0.8)^2}{2l} = \frac{32 \times 1.2^2}{2 \times 0.6} = \underline{\underline{28.8\,J}}$

$v=0$, so no E_k.

"Missing energy" is equal to work done against friction

so use $Wd = Fs$. with $s = 5$ (distance)

$Wd = 96.8 - 28.8 = 68\,J$

so $68 = F \times 5 \Rightarrow F = \frac{68}{5} = \underline{\underline{13.6\,N}}$

(c) $T = \frac{\lambda x}{l} = \frac{32 \times 1.2}{0.8} = 48\,N$

$48 > 13.6$ so Tension > friction $\therefore$ will start to move.

(d) Slack and at rest so $E_k = 0$ and $EPE = 0$.

all EPE at point B is used as work against friction.

Let distance $BC = y$, then use $Wd = Fs$

$28.8 = 13.6y \Rightarrow y = \frac{28.8}{13.6} = 2.11765$

distance $= 2.12\,m$ (3SF)

(e) distance $= 5 + 2.11765 = 7.12\,m$ (3SF).